Modelling of Equipment Loaded Panels for Robust Control Investigations

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Abstract
This paper develops a modelling technique for equipment load panels which directly produces (adequate) models of the underlying dynamics on which to base robust controller design/evaluations. This technique is based on the use of the Lagrange's equations of motion and the resulting models are verified against those produced by a finite Element Method Model.

1. Introduction

Over the last twenty years there has been a dramatic increase of research in the field of active control of vibrations with particular interest in aerospace applications. One of the main streams of research has focused on Large Space Structures (LSS), [1] which are characterised by very low rigidity and damping, with the objective of reducing the large oscillations of the structure produced by various dynamic disturbances or manoeuvres. In particular the large model uncertainties which characterise LSS have been the object of study and pose challenging problems for the control community. Several controller algorithms, implemented using different types of sensors and actuators, have been studied and many different techniques for the optimal design of the controller and structure have been proposed. To support this theoretical work, ground based hardware tests have been performed in some cases as a prelude to actual implementation in space.

The onward development of real LSS, has been significantly hindered by the delays in the plans to build the Space Station. This stagnation of research in this particular application area has, in turn, led to increased research effort on more realistic and immediate issues [2]. These include active control of microvibrations on satellites structures which is now quite a vibrant area. For example, mirror pointing systems, such as those used in space telescopes which require extremely accurate targeting, are very sensitive to vibrations and even small mechanical disturbances can produce jitter resulting in the blurring of images. It is estimated that the new generation of precision pointing mirror systems will be extremely sensitive to even the smallest vibrations and to exploit their capability a decrease in the satellite jitter of three orders of magnitude would be required. In other equipment, such as laser communication systems these vibrations induce an oscillation of the beam that can cause problems at the receiving station

The starting point for investigations into different strategies for the reduction of the vibration level at the sensitive equipment location, is the development of suitable models to simulate the dynamics of the system and, in particular, it is desirable to have models able to represent the whole vibration path, from source to receiver. These models ought to capture the essential dynamics of the structures response in a form suitable for active control design studies, i.e. a state space model or description.

The propagation of vibrations along the structure between the source and the receiver is influenced by changes in many parameters, e.g. details of geometrical configurations and material properties. Most of these parameters, which are included in the mathematical model can have values which are not known precisely and there could also be other variables or disturbances which are not included in the plant model for various reasons, but still influence the vibration transmission in the real structure. Large model uncertainties are therefore an important issue and several robust control algorithms, some of which have been developed and applied to LSS, are available for application to the structural model of the plant.

Active control strategies require a suitable mathematical model of the system dynamics. This paper develops a general strategy for the production of models on which to base robust controller design strategies based on, for example H_∞ or Loop Transfer Recovery methods which are now very well established in the control system community.

2. Plant model

2.1 Plant description

The three characteristic elements of a vibration transmission problem are respectively:

- source(s)
- transmitting structure
- receiver(s)

also if active control is to be implemented, sensors and actuators have to be included in the plant model. The mathematical model of the plant considered in this paper, is derived using the Lagrange's equations of motion.

The plant modelled is a rectangular panel on which equipment boxes are mounted. These act as source(s)/receiver(s) or as passive elements.

2.2 Source(s) and receiver(s)

Source(s) and receiver(s) are usually mounted on suspensions, whose basic function is to secure the piece of equipment to the underlying structure and at the same time minimise the transmission of dynamic disturbances i.e. vibrations. These two requirements are in principle contradictory, since the first one produces strong coupling between the equipment and the underlying structure dynamics, while to meet the second requirement it is necessary to isolate the equipment, thus uncoupling it from the supporting structure. This contradiction can be solved by using latching mechanisms, which are locked when the equipment needs to be strongly constrained to the structure, e.g. the launch phase of a satellite, and open during phases in which the equipment needs to be isolated, e.g. equipment for microgavity experiments. The complications of using a two mode suspension system can be avoided by having suspensions which are stiff at low frequency, thus allowing the transmission of low frequency loads, such as those produced by manoeuvres or proper static loads, and soft at high frequency, in order to avoid the transmission of dynamic disturbances (which are usually occur at higher frequencies). The suspension therefore acts as a low pass filter and provided that the bandwidth of the mechanical disturbances is at a frequency high enough to be well separated from the deliberately transmitted loads, then the problem could be solved by passive suspensions.

Conversely, there are many uncertainties in the system and this fact, together with the high performance level required, means that active control, possibly implemented in a closed-loop configuration must be employed. This is the only real option available to achieve the required performance levels in the presence of unknown disturbances. The first task in this strategy is to construct a mathematical model of the particular structure under consideration as discussed next.

In terms of sources, these are basically pieces of equipment where the motion of the internal parts to perform their function produces unwanted forces/reactions which are transmitted to the supporting structure. Each particular piece of equipment has a different mechanical interface to secure it to the structure underneath, the most common mounting geometry being four mounting feet positioned at the corners of the equipment enclosure. For this reason a typical piece of equipment is modelled as if it were connected to the underlying structure through mounting feet, positioned at its corners (see Figure 1).

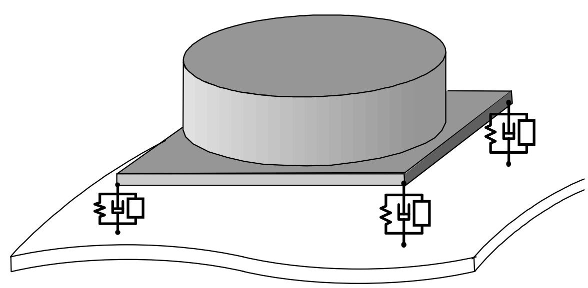


Figure 1: Equipment with active suspensions mounted on a flexible panel.

Since the mechanical enclosure of the equipment is quite stiff, compared to the panel, it can be modelled as a rigid body connected to a panel through flexible elements, which represent the mounting feet. The description of dynamics of a free rigid body in a three dimensional space requires six degrees of freedom (dof), which are usually taken as three translations and three rotations relative to a fixed reference system. In this case the in plane translations (along x and y axes) of the enclosure can be neglected, assuming strong mechanical coupling with the panel along these directions and the high in plane stiffness of the panel itself. Rotations about the axis (z) perpendicular to the panel are transmitted to the panel as x and y translations of the mounting feet and therefore can be neglected for the same reasons as those given above.

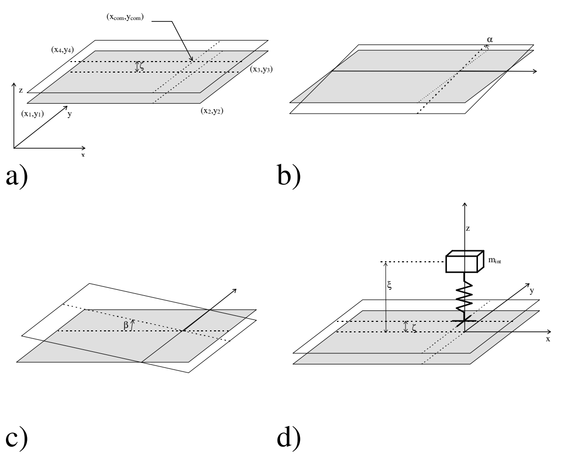


Figure 2: Degrees of Freedom of the equipment enclosures

Each enclosure is therefore assumed to have only three degrees of freedom, i.e. off-plane displacement ζ and pitch and roll angles α and β, as illustrated by Figures 2 a), b) and c).

Most of the equipment, even though included in quite stiff enclosures (such as electronic boxes), will have internal flexible parts whose resonance could be within the frequency bandwidth of the active control system. It is therefore necessary to account for these internal dynamics and, by assuming only one resonance within the frequency range of interest, the internal dynamics can be reproduced by adding an extra dof (ξ) to the set of three so far used to describe the equipment motion.

This new dof describes the z displacement of an internal mass linked through a spring to the Centre of Mass (CoM) of the equipment enclosure, as shown in Figure 2 d). This internal resonator can be tuned to the internal resonance frequency of the equipment by varying its mass and spring stiffness.

Given these assumptions it is possible to write suitable expressions for the energy associated with the equipment as a function of its degrees of freedom (which are taken as generalised co-ordinates). The expressions for the energies obtained will then be derived and Lagrange's equations of motion applied, to obtain the mathematical model of the dynamics of the plant.

2.2.1 Kinetic Energy

Suppose that $\boldsymbol{\phi}_{eqp}$ denotes the column vector of the equipment degrees of freedom:

$$\boldsymbol{\phi}_{eqp} = \{\alpha \quad \beta \quad \zeta \quad \xi\}^t \quad (1)$$

and $\mathbf{M}_{eqp}$ the inertia matrix:

$$\mathbf{M_{eqp}} = diag\{I_{yy} \quad I_{xx} \quad m_{enc} \quad m_{int}\} \quad (2)$$

where I_{yy} and I_{xx} are the moments of inertia of the equipment, m_{int} is the internal mass i.e. the mass of the resonating element, and m_{enc} is the mass of the equipment enclosure. Then it is possible to write the kinetic energy as:

$$\mathrm{T_{eqp}} = \frac{1}{2}\dot{\boldsymbol{\phi}}^t_{eqp}\mathbf{M}_{eqp}\dot{\boldsymbol{\phi}}_{eqp} \quad (3)$$

2.2.2 Potential Energy

Assuming that the equipment enclosure can be regarded as a rigid body on comparatively soft suspensions, the potential energy is essentially the energy stored in the suspension system, plus the energy stored in the 'spring' of the internal resonator. Modelling the suspension system (Figure 1) as a parallel connection of a spring, a damper and an active element, i.e. a piezoelectric prism whose stiffness is included in the spring stiffness, the elastic energy stored in the strained suspension can be expressed as:

$$\mathrm{U^{ela}_{eqp}} = \frac{1}{2}(k_s\Delta \mathrm{z}_\mathrm{i}^2 + k_{res}\Delta \mathrm{z}^2_\mathrm{res}) \quad (4)$$

where k_s is the total stiffness of the suspension, k_{res} the stiffness of the internal resonator, Δz_i, i = 1,…,4, is the linear deformation of the i-th suspension foot and Δz_{res} is the linear deformation of the internal resonator stiffness. The linear deformation of the i-th suspension can be obtained as the difference between the out-off-plane displacement of the panel surface $z_{str}(x_i,y_i,t)$ evaluated at the mounting foot location (x_i,y_i) and the vertical displacement $z_i(t)$ of the i-th corner of the box. Both of these displacements can be written in terms of shape functions **s**, multiplied by time dependent co-ordinates $\boldsymbol{\phi}$ as:

$$(z_{str})_i = (\mathbf{s}_{str})_i^t \boldsymbol{\phi}_{str} \quad (5)$$

and

$$z_i = (\mathbf{s}_{eqp})_i^t \boldsymbol{\phi}_{eqp} \quad (6)$$

The shape functions used for the supporting structure will be described in the following section, while those used for the equipment are simply:

$$(\mathbf{s}_{eqp})_i^t = \{(y_i - y_{CoM}) \quad (x_i - x_{CoM}) \quad 1 \quad 0\}^t \quad (7)$$

Hence the linear deformation can be written as:

$$\Delta z_i = \{(\mathbf{s}_{str})_i^t \quad -(\mathbf{s}_{eqp})_i^t\}\begin{Bmatrix}\boldsymbol{\phi}_{str} \\ \boldsymbol{\phi}_{eqp}\end{Bmatrix} \quad (8)$$

and the deformation of the spring of the internal resonator can be written as:

$$\Delta z_{res} = (\mathbf{s}_{res})^t \boldsymbol{\phi}_{eqp} \quad (9)$$

with:

$$\mathbf{s}_{res}^t = \{0 \quad 0 \quad 1 \quad -1\}^t \quad (10)$$

Consequently the total elastic energy associated with any piece of equipment is:

$$U_{eqp}^{ela} = \frac{1}{2}\boldsymbol{\phi}^t \mathbf{K}_{eqp}^{ela} \boldsymbol{\phi} \quad (11)$$

where:

$$\boldsymbol{\phi} = \begin{Bmatrix}\boldsymbol{\phi}_{str} \\ \boldsymbol{\phi}_{eqp}\end{Bmatrix} \quad (12)$$

and

$$\mathbf{K}_{eqp}^{ela} = k_s \sum_{i=1}^{4}\begin{bmatrix}(\mathbf{s}_{str})_i(\mathbf{s}_{str})_i^t & -(\mathbf{s}_{str})_i(\mathbf{s}_{eqp})_i^t \\ -(\mathbf{s}_{eqp})_i(\mathbf{s}_{str})_i^t & (\mathbf{s}_{eqp})_i(\mathbf{s}_{eqp})_i^t\end{bmatrix} + k_{int}\begin{bmatrix}0 & 0 \\ 0 & \mathbf{s}_{res}\mathbf{s}_{res}^t\end{bmatrix} \quad (13)$$

The elastic energy is not the only potential energy which can be stored in the suspensions. In particular the stress produced by the piezoelectric effect, multiplied by the strain of the material and integrated over the volume of the piezoelectric element produces another form of energy, termed elastoelectric energy here.

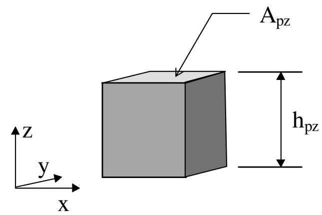


Figure 3: Piezoelectric prism

The stress produced in a piezoelectric prism along the direction (z in the reference system used here) of the applied electric field is given by the expression:

$$(\sigma_{elect})_i = E_{pz} d_{zz} \frac{v_i}{h_{pz}} \quad (14)$$

where d_{zz} is the piezoelectric constant of the material, h_{pz} is height of the prism, E_{pz} is the Young's modulus and v_i. is the applied voltage. Assuming that the strain in the material is constant and equal to:

$$(\varepsilon_z)_i = \frac{\Delta z_i}{h_{pz}} \quad (15)$$

then the elastoelectic energy stored in each piezoelectric support is:

$$(U_{eqp}^{elastoelec})_i = \frac{1}{2}\iiint_{pz}(\sigma_{elect})_i(\varepsilon_z)_i \, dxdydz = \frac{1}{2} v_i E_{pz} d_{zz} \frac{A_{pz}}{h_{pz}} \Delta z_i$$
(16)

Letting $\mathbf{v}_{eqp}$ be the column vector of the voltages v_i existing at the electrodes of each mounting foot, the total elastoelectric energy stored in the equipment suspension system can be written as:

$$U_{eqp}^{elastoelect} = \mathbf{v}_{eqp}^{t} \boldsymbol{K}_{eqp}^{elastoelect} \boldsymbol{\phi} \quad (17)$$

where

$$\boldsymbol{K}_{eqp}^{elastoelect} = E_{pz} d_{zz} \frac{A_{pz}}{2h_{pz}} \begin{bmatrix} \left\{ (\mathbf{s}_{str})_1^t \quad -(\mathbf{s}_{eqp})_1^t \right\} \\ \vdots \\ \vdots \\ \left\{ (\mathbf{s}_{str})_4^t \quad -(\mathbf{s}_{eqp})_4^t \right\} \end{bmatrix} \quad (18)$$

under the assumption that all supports have the same characteristics.
The last form of energy stored in the piezoelectic material is the electric energy which can be calculated directly as:

$$(U_{eqp}^{elect})_i = \frac{1}{2} C v_i^2 \quad (19)$$

where C is the capacitance of the elements given by:

$$C = \varepsilon_{pz} \frac{A_{pz}}{h_{pz}} \quad (20)$$

Hence the total electric energy stored can be written in matrix form as:

$$U_{eqp}^{elect} = \frac{1}{2} \mathbf{v}_{eqp}^{t} \mathbf{K}_{eqp}^{elect} \mathbf{v}_{eqp} \quad (21)$$

where $\mathbf{K}_{eqp}^{elect}$ is the four by four diagonal matrix whose diagonal elements are all equal to the capacitance.

2.2.3 Dissipative forces

The dissipative forces produced by dashpots in the mounting feet can be calculated as an extra term in the generalised forces which will represent the externally applied disturbance forces. This term, denoted by Q_{dp} here, accounts for the non conservative forces:

$$F_i = c\Delta \dot{z}_i \quad (22)$$

generated by each of the i (i = 1,…,4) dashpots of the mountings. Expression (8) in this case can be re-written in matrix form as:

$$F_i = c\left\{ (\mathbf{s}_{\mathbf{str}})_i^t \quad -(\mathbf{s}_{\mathbf{eqp}})_i^t \right\} \begin{Bmatrix} \dot{\boldsymbol{\phi}}_{str} \\ \dot{\boldsymbol{\phi}}_{\mathbf{eqp}} \end{Bmatrix} \quad (23)$$

and adopting the sign convention of Figure 4,

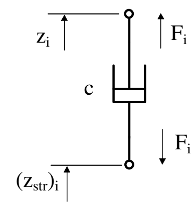


Fig 4: Dashpot

it is possible to write the vector of the generalised forces acting on the panel and on the equipment respectively as:

$$\mathbf{Q}_{str} = F_i (-\mathbf{s}_{str})_i \quad (24)$$

$$\mathbf{Q}_{eqp} = F_i (\mathbf{s}_{\mathbf{eqp}})_i \quad (25)$$

By assumption, the equipment has four mounting feet and equations (23), - (25) can be used to write the generalised force vector as:

$$\mathbf{Q}_{\mathbf{dp}} = \mathbf{C}_{\mathbf{dp}} \dot{\boldsymbol{\phi}} \quad (26)$$

where:

$$\mathbf{C}_{\mathbf{dp}} = c\sum_{i:1}^{4} \begin{bmatrix} (\mathbf{s}_{\mathbf{str}})_i (\mathbf{s}_{\mathbf{str}})_i^t & (-\mathbf{s}_{\mathbf{str}})_i (\mathbf{s}_{\mathbf{eqp}})_i^t \\ (-\mathbf{s}_{\mathbf{eqp}})_i (\mathbf{s}_{\mathbf{str}})_i^t & (\mathbf{s}_{\mathbf{eqp}})_i (\mathbf{s}_{\mathbf{eqp}})_i^t \end{bmatrix} \quad (27)$$

2.2.4 Extension to *Ne* pieces of Equipment

Using the procedure developed here, it is possible to write expressions for the energies associated with all pieces of equipment mounted on the panel which meet the criteria of section 2.2. The matrices produced will then be partitioned to separate the dependence on the panel and equipment boxes degrees of freedom and expanded and assembled in a FEM fashion. As a typical example, equation 28 shows the partitioning of the matrix $\mathbf{K}_{eqp}^{ela}$

$$\mathbf{K}_{eqp}^{ela} = \left[\begin{array}{c|c} \mathbf{K}_{pp} & \boldsymbol{K}_{pe} \\ \hline \boldsymbol{K}_{ep} & \mathbf{K}_{ee} \end{array} \right] \quad (28)$$

and considering *Ne* pieces of equipment, the total stiffness matrix produced by the elasticity of the equipment mountings and internal resonators will be:

$$\mathbf{K}_{eqp}^{ela} = \sum_{j:1:}^{Ne} \left[\begin{array}{c:ccc} (\mathbf{K}_{pp})_j & \cdots & (\boldsymbol{K}_{pe})_j & \cdots \\ \hdashline \vdots & \ddots & & 0 \\ (\boldsymbol{K}_{ep})_j & & (\mathbf{K}_{ee})_j & \\ \vdots & 0 & & \ddots \end{array}\right] \quad (29)$$

2.3 Modelling the transmitting structure

The transmitting structure consists of a simply supported panel, on which the pieces of equipment are mounted. Lumped masses can be positioned on the panel to represent equipment with dimensions much smaller than the panel deformation wavelength. Piezoelectric patches bonded on the faces of the panel act as sensors and actuators for the control system and are used to apply control along the transmitting structure. The modal shapes of the bare panel are used as the modal shapes of the transmitting structure.

2.3.1 Transmitting structure energies

The energy expressions are derived using the procedure described in [3], which consists of writing expressions for the kinetic and potential energy associated with all the elements of the transmitting structure. The kinetic energies associated with the panel, the piezoelectric patches and the lumped masses can be written as:

$$\mathrm{T}_{\mathrm{pl}} = \frac{1}{2}\dot{\boldsymbol{\phi}}_{\mathrm{str}}^{\mathrm{t}}\mathbf{M}_{\mathbf{pl}}\dot{\boldsymbol{\phi}}_{\mathrm{str}}, \qquad \mathrm{T}_{\mathrm{pz}} = \frac{1}{2}\dot{\boldsymbol{\phi}}_{\mathrm{str}}^{\mathrm{t}}\mathbf{M}_{\mathbf{pz}}\dot{\boldsymbol{\phi}}_{\mathrm{str}}$$

$$\mathrm{T}_{\mathrm{lm}} = \frac{1}{2}\dot{\boldsymbol{\phi}}_{\mathrm{str}}^{\mathrm{t}}\mathbf{M}_{lm}\dot{\boldsymbol{\phi}}_{\mathrm{str}} \quad (30)$$

and the potential energies as:

$$\mathrm{U}_{\mathrm{plate}} = \frac{1}{2}\boldsymbol{\phi}^{\mathrm{t}}\mathbf{K}_{\mathbf{pl}}\boldsymbol{\phi} \qquad \mathrm{U}_{\mathrm{pz}}^{\mathrm{elast}} = \frac{1}{2}\boldsymbol{\phi}^{\mathrm{t}}\mathbf{K}_{\mathbf{pz}}^{\mathbf{elast}}\boldsymbol{\phi}$$

$$\mathrm{U}_{\mathrm{pz}}^{\mathrm{elastelect}} = \mathbf{v}_{\mathrm{strt}}^{t}\boldsymbol{K}_{\mathrm{pz}}^{\mathrm{elastelect}}\boldsymbol{\phi} \quad \mathrm{U}_{\mathrm{pz}}^{\mathrm{elect}} = \frac{1}{2}\mathbf{v}_{str}^{t}\mathbf{K}_{\mathbf{pz}}^{\mathbf{elect}}\mathbf{v}_{str}^{t} \quad (31)$$

Note also that no potential energy is associated with the lumped mass.

2.4 Equations of motion

Suppose now that all the energy functions are available as functions of the supporting structure and equipment dof, i.e. $\boldsymbol{\phi}_{\mathrm{str}}$, $(\boldsymbol{\phi}_{\mathrm{eqp}})_j$, $\mathbf{v}_{\mathrm{str}}$ and $(\mathbf{v}_{\mathrm{eqp}})_j$, then with these vectors as generalised co-ordinates $\mathbf{q}_i$, the application of Lagrange's equations of motion, i.e.

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{T}}{\partial \dot{\mathrm{q}}_{\mathrm{i}}}\right) - \frac{\partial \mathrm{T}}{\partial \mathrm{q}_{\mathrm{i}}} + \frac{\partial \mathrm{U}}{\partial \mathrm{q}_{\mathrm{i}}} = \mathrm{Q}_{\mathrm{i}} \quad (32)$$

is quite straight forward. Here the generalised forces $\mathbf{Q}_i$ represent the externally applied forces and the forces produced by the dashpots in the suspension.
Using the mechanical dofs, assembled in a vector $\boldsymbol{\phi}$, in Lagrange's equations of motion yields a 2nd order matrix differential equation of the form:

$$\mathbf{M}\ddot{\phi} + \mathbf{C}\dot{\phi} + \mathbf{K}\phi = \mathbf{Vv} + \mathbf{Sf} \quad (33)$$

This describes system dynamics when driven by the actuator voltages, $\mathbf{v}_{\mathrm{str}}$ and $(\mathbf{v}_{\mathrm{eqp}})_j$ which have been assembled into a vector $\mathbf{v}$, and the external forces are represented by the vector $\mathbf{f}$.
Applying Lagrange's equations of motion with the electrical dof as generalised co-ordinates yields a second set of equations, relating mechanical and electrical dof. Also this equation can be used to express the electrical signals as functions of the mechanical dof obtained solving equation (33).
This yields the following equation:

$$\mathbf{v}_s = -\left(\mathbf{K}_s^{\mathbf{elect}}\right)^{-1}\boldsymbol{K}_s^{\mathbf{elastoelect}}\boldsymbol{\phi} \quad (34)$$

which can, for example, be used to compute the voltages at the patches acting as sensors (the elements of the vector $\mathbf{v}_s$ of (34)).

3. Model verification

Model verification has been performed by comparing the results obtained using the modelling technique described in this paper against those obtained by modelling the same structure with the Finite Element Method (FEM). More details on this aspect are given next.

3.1 Transmitting structure verification

The first set of tests aimed to validate the model of the transmitting structure. Figure 5 shows the FEM model of the structure and Figure 6 shows the deformed shapes of the structure when subjected to 1 N force disturbance at 400Hz. Figure 7 shows a comparison of the frequency response of the FEM model (dashed line) and the Lagrange model (continuos line). These results are typical in that they demonstrate a high level of agreement between the two models.

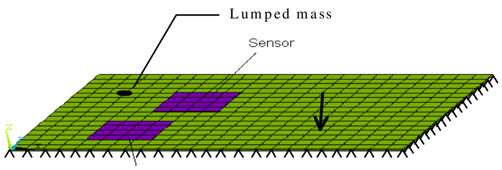
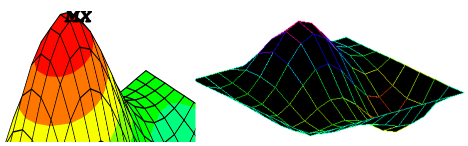


Figure 5: FE model of the transmitting structure.



FE Model Lagrange Model

Figure 6: Deformed shape at 400 Hz

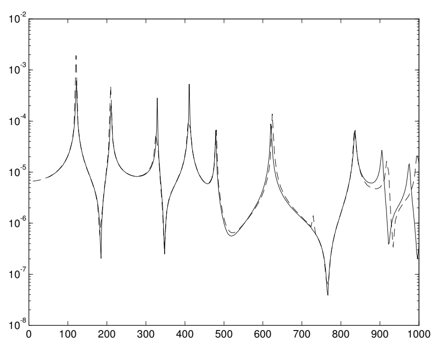


Figure 7: Displacement at the centre of the panel produced by 1 N point force.

3.2 Complete plant verification

The verification of the model of the whole assembly has been undertaken by comparing the output produced by a moment applied on the equipment box acting as a source.

Figure 8 shows a schematic view of the assembly and in Figure 9 and 10, comparisons of the plant response to different inputs are given. From these plots it is clear that agreement is not very good at high frequency where the frequency predicted by the FEM are higher than those produced by the Lagrange model. Comparisons made using more refined FE models have concluded that the differences can be reduced using finer meshes.

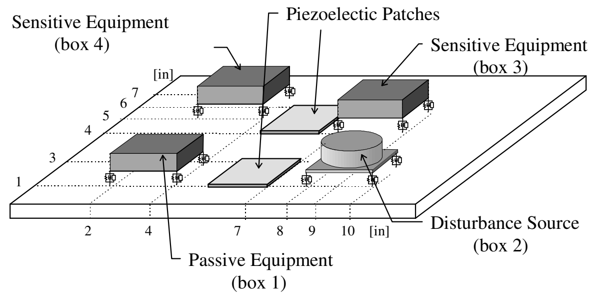


Figure 8: Model assembly

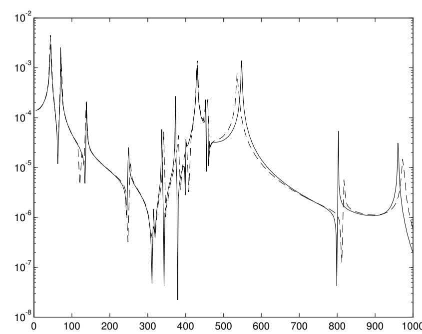


Figure 9: Displacement of receiver 1 produced by moment on the source.

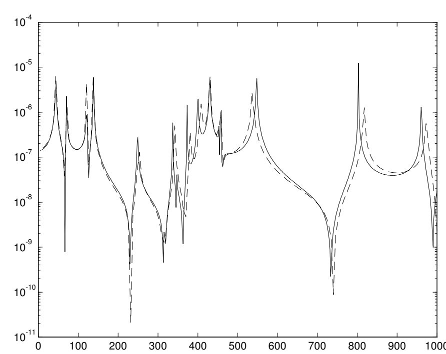


Figure 10: Displacement of receiver 1 produced by active suspension input.

4. Control System Formulation

Once the mathematical model of the plant dynamics is available, the next step is to write it in a form suitable for active control studies. In particular, it is a straight forward task to write this model in state space form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{\Gamma}\mathbf{w}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} \qquad (35)$$

where:

$$\mathbf{x} = \begin{pmatrix} \boldsymbol{\phi} \\ \dot{\boldsymbol{\phi}} \end{pmatrix} \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{V} \end{bmatrix} \quad \mathbf{\Gamma} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{S} \end{bmatrix} \quad \mathbf{w} = \mathbf{f} \qquad \mathbf{u} = \mathbf{v}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{s}_{\mathbf{out}}^{\mathbf{t}} & 0 \\ \left(-\mathbf{K}_{\mathbf{s}}^{\mathbf{elect}}\right)^{-1} \boldsymbol{K}_{\mathbf{s}}^{\mathbf{elastelect}} & 0 \end{bmatrix} \quad (36)$$

The input dynamic disturbances term **f** represents forces and moments applied at the source(s) or directly at specified locations on the panel, whilst the other inputs which form the vector **u** are the commands to the actuators, which can either be the suspension system of source(s) and/or receiver(s), or/and the piezoelectric patches bonded on the panel.

As output(s) the displacement at any position on the panel and/or on the receiver(s) are available, together with the signals generated by the sensors. The aim of the control system, F in the block diagram of Figure 11, is to minimise the displacement at selected location(s) in presence of the dynamic disturbances w and of uncertainties in the plant model Δ.

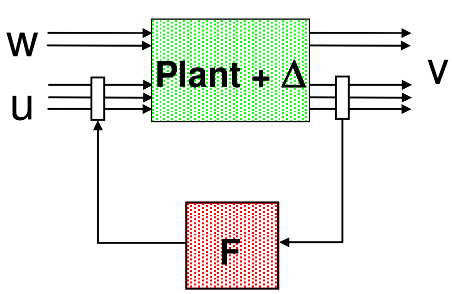


Figure 11: Plant

This is the standard form for robust control design/investigation studies using, say, H_∞ or LTR methods. In depth research on the design and evaluation of such controllers is currently in progress with preliminary results reported in [4].

5. Conclusions

This paper has used Lagrange's equations of motion to develop state space models of equipment loaded panels on which to undertake the design of active control strategies based on robust feedback control schemes. The basis of this work is a mass loaded panel on which various pieces of equipment are mounted. The equipment has been modelled as rigid rectangular boxes mounted on a flexible panel. The enclosures have internal resonators to simulate internal dynamics and there is provision for rigid or flexible mounting elements to allow for active/passive suspensions.

Piezoelectric patches are assumed as sensors and actuators on the panel and piezoelectric stacks are used for the box suspensions. Before controller design can begin, however, it is clearly necessary to establish that the model of any particular configuration is an 'adequate representation' of its dynamics. Here the use of FE models has been strongly supported by case studies - with a more detailed treatment in [4].

Currently in depth controller design/evaluations are being undertaken with preliminary results in [5]. A more detailed treatment of this work will be reported in due course.

References

1. G.S. Nurre, R.S. Rryan, H.N. Scofield and J.L. Sims *Dynamics and Control of Large Space Structures*, J. of Guidance & Control, Vol.7, N 4, pp. (1984).
2. H.R. Stark and C. Stavrinidis, *ESA Microgravity And Microdynamics Activities - An Overview,* Acta Astronautica 34, pp. 205-221, (1994).
3. G.S.Aglietti, S.B.Gabriel, R.S.Langley and E.Rogers, *A Modelling Technique For Active Control Design Studies With Application To Spacecraft Microvibrations***,** J. of The Acoustical Society of America, 102 (4), pp 2158-2166, (1997).
4. G.S. Aglietti, R.S.Langley, E.Rogers and S.B.Gabriel, Research Report, Dept. of Aeronautics and Astronautics, University of Southampton, UK, (1998)
5. G.S.Aglietti, R.S.Langley, E.Rogers and S.B.Gabriel, *Active Control Strategies for Equipment Loaded Panels*, MOVIC '98 - The Fourth International Conference on Motion and Vibration Control, Zurich Switzerland, August (1998)