

Gain-scheduled Linear Quadratic Control of Wind Turbines Operating at High Wind Speed

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Abstract—This paper addresses state estimation and linear quadratic (LQ) control of variable speed variable pitch wind turbines. On the basis of a nonlinear model of a wind turbine, a set of operating conditions is identified and a LQ controller is designed for each operating point. The controller gains are then interpolated linearly to get a control law for the entire operating envelope.

The states and the gain-scheduling variable are not online available and an observer is designed. This is done in a modular approach in which a linear estimator is used to estimate the non-measured state variables and the unknown input, aerodynamic torque. From the estimated aerodynamic torque and rotor speed and measured pitch angle the scheduling variable effective wind speed) is calculated by inverting the aerodynamic model.

Simulation results are given that display good performance of the observers and comparisons with a controller designed by classical methods display the potential of the method.

I. INTRODUCTION

Within the past decade the cost-effectiveness of wind turbines has increased significantly. This is primarily achieved by reducing the amount of material for a fixed wind turbine size. This reduction of mass in the structural components causes each component to be less robust towards fatigue loads. The problem can be countered by the introduction of active control, which was done quite some years ago.

In this paper we consider wind turbines for which it is possible to rotate the blades along the longitudinal axis (denoted pitch) and hereby controlling the energy input. Furthermore we consider a wind turbine with a doubly-fed induction generator with which it is possible to control the reaction torque from the generator to make the rotational speed vary approximately $\pm 30\%$ from the synchronous speed. This way we can better control loads in the transmission system and also obtain a higher energy output at low wind speeds. In the following we will denote such a wind turbine: a variable speed, variable pitch wind turbine.

This introduction of two control variables (pitch angle and generator torque) has led to many investigations in the design of control algorithms that give the best trade-off between variations in the power and fatigue loads. Also the LQ control technique has been applied to the control of wind turbines. [1], [2], [3], [4]

These publications address the design of a static state feedback controller for a linearised plant model at a selected wind speed, and most of them also address the problem of estimating the states using Kalman filters or similar. Also the

problem of interpolating the controllers has been addressed in [4] in which the gain scheduling variable is estimated from steady state equations.

The papers in the literature on LQG control of wind turbines are in general split into three different categories: Some present a detailed controller design for a model linearized at a single operating point. The two other categories focus on the nonlinear model but take very different approaches: either the academic approach with focus on modern control techniques or the practical approach which focuses on getting simple algorithms working in practice. This means that focus is in most papers either on the application of modern control techniques with the relation to physical requirement missing to some degree. The opposite approach with the large focus on physical interpretation often misses the possible advantages of modern control techniques.

This paper is an attempt to close the gap between the papers focusing on modern control techniques and the focus on requirements that are met in today's operation of wind turbines. A gain-scheduled LQ controller is designed with performance weight similar to that of [3] – with the addition of a weight on the shaft torque to limit fatigue loads in the transmission system. For the state estimation it should be noted that the disturbance (wind speed) has a large impact on the wind turbine dynamics and that this disturbance is not measured. Because of this it has been chosen to use the principles of disturbance estimation [5] in the observer design. Furthermore the wind speed will be used as the gain scheduling variable and must therefore be online available.

II. WIND TURBINE MODEL

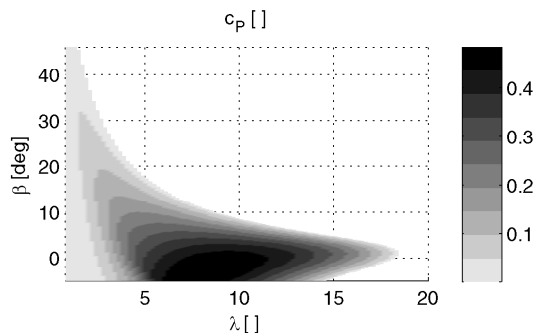
In this control formulation we are interested in maintaining the generator speed within its limits, minimising the power fluctuations around its nominal value, and keeping the fatigue loads in the transmission below a certain level. The controller design will be based on a two degree of freedom model of the transmission, a static model of the aerodynamics and actuator dynamics.

A. Aerodynamics

The main input to the wind turbine is the wind, which through the aerodynamic lift and drag effects the main shaft by a driving torque. The angle of attack of the wind onto the blades can be assumed dependent on only the pitch

orientation of the blades and the ratio between the speed of the blade tip and the wind speed (denoted tip-speed-ratio). We assume for simplicity that all blades are pitched to the same orientation and that the lift/drag on the blades directly affect the driving torque, Q_a , on the main shaft – i.e. the structural dynamics of the blades are incorporated into the parameters of the model of the transmission system. With this assumption we can describe the aerodynamics as a static nonlinear mapping of the collective pitch orientation, β , the angular velocity of the rotor, ω_r , and the effective wind speed¹, v , as shown in (1) where ρ is the air density and R is the rotor radius. The function c_P describes the aerodynamic efficiency of the rotor design and is described by a nonlinear mapping of the pitch angle and tip speed ratio, λ , as illustrated in Fig. 1

$$Q_a = \frac{1}{2} \rho \pi R^2 \frac{v^3}{\omega_r} c_P(\beta, \lambda), \quad \lambda = \frac{R \omega_r}{v} \quad (1)$$



1: Illustration of aerodynamic coefficient, c_P .

B. Transmission system

The aerodynamic torque, Q_a , from (1) is input to the transmission system on the low speed side. The transmission system is modelled as two inertias interconnected by a spring/damper and a gearing. The rotor inertia and stiffness is included in the parameters of the low speed side. On the high speed side the generator is mounted giving opportunity to control the reaction torque from the generator. Friction, stiffness, etc. is assumed linear leading to a linear model of the transmission system with aerodynamic torque, Q_a , and generator torque, Q_g , as inputs. The outputs are angular speed on the low speed side, ω_r , on the high speed side, ω_g , and the torsion between the two inertias, Q_{sh} . From these observations we can setup a dynamic model of the transmission system of the form (2) with x_t being the state vector.

$$\dot{x}_t = A_t \cdot x_t + B_{t,r} \cdot Q_a + B_{t,g} \cdot Q_g \quad (2a)$$

$$Q_{sh} = C_{t,Q} \cdot x_t \quad (2b)$$

$$\omega_r = C_{t,r} \cdot x_t \quad (2c)$$

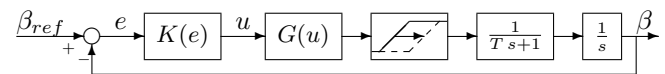
$$\omega_g = C_{t,g} \cdot x_t \quad (2d)$$

¹The effective wind speed (also denoted the free wind speed) is an abstract term that describes the spatial average of the wind field at the rotor position with the wind stream not being affected by the wind turbine.

C. Pitch system

To deal with the low frequency variations in wind speed we can alter the pitch orientation of the blades causing the aerodynamic torque to be manipulated. The actuator is highly nonlinear and a cascade coupled solution has been chosen to handle the nonlinearity.

In Fig. 2 the loop containing both the model of the pitch actuator and its associated controller is shown. The actuator is a hydraulic actuator with the transfer function from control voltage to pitch rate being modelled as a combination of a static nonlinear gain, $G(u)$, a time delay and a low pass filter. To counteract the nonlinear gain, $G(u)$, a gain-scheduled proportional controller, $K(e)$, has been designed to track a pitch reference, β_{ref} .



2: Inner pitch control loop

From the perspective of the outer loop, the proportional controller, $K(e)$, has a linearising effect on the nonlinear pitch gain when the pitch error, e , is in the region of nominal operation. For larger errors the nonlinearity has still quite some effect which means that we cannot use it in extreme operating conditions that lead to extraordinary pitch activity. This can happen in extreme weather conditions or in the case of faults in the wind turbine leading to very high pitch activity. However, for nominal operation we conclude that the linear model is inappropriate.

D. Generator and converter system

On the high speed shaft we can change the reaction torque from the generator by changing the ratio between power in the rotor and stator. This control action enables the possibility to make a trade-off between the variations in active power production and variations in rotational speed.

The generator and converter dynamics is modelled as a constant gain because it only contains very high frequency components. Then the active power, P_e , can be expressed as the sum of the power in the rotor, P_r and stator, P_s . Furthermore the power in the rotor is proportional to the stator power and the slip, $s = \frac{1}{\omega_{NET}}(\omega_g \cdot pp - \omega_{NET})$ – with pp being the number of pole pairs in the generator. This means that the electric power can be expressed as in (3).

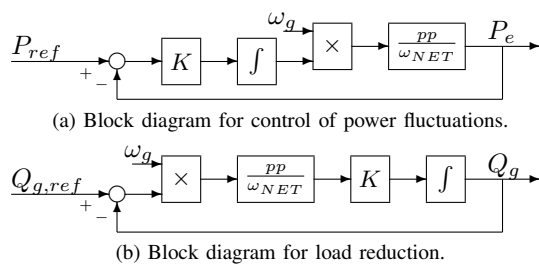
$$\begin{aligned} P_e(t) &= P_s(t) + P_r(t) = P_s(t) \cdot (1 + s(t)) \\ &= P_s(t) \cdot \left(1 + \frac{\omega_g(t) \cdot pp - \omega_{NET}}{\omega_{NET}} \right) \\ &= \frac{pp}{\omega_{NET}} \cdot P_s(t) \cdot \omega_g(t) \end{aligned} \quad (3)$$

The loss in the generator and converter is assumed proportional to the active power and independent of the operating condition. This means that the reaction torque from the generator, Q_g , can be expressed as in (4) with η being the

generator/converter efficiency.

$$Q_g(t) = \frac{P_e(t)}{\eta \cdot \omega_g(t)} = \frac{pp}{\eta \cdot \omega_{NET}} \cdot P_s(t) \quad (4)$$

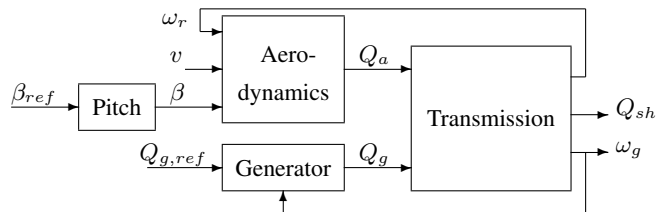
The main objective of the controller design for the generator loop is to ensure a proper power quality and to produce the desired power level. The controller includes high frequency components as well as integral action on the tracking of the power reference. The high frequency components can be disregarded when seen from the outer loop which leads to the loop illustrated in Fig. 3a with K being a linear control gain. When designing the outer loop, it is more relevant to have the formulation in terms of the reaction torque which can be achieved by using (4). Then we get a generator closed loop model as in Fig. 3b.



3: Block diagram of generator loop.

E. Interconnection

To summarise, the model of the wind turbine consists of four components: A static nonlinear function describing the aerodynamics, a third order LTI model of the transmission system, and a gain-scheduled first order model of the two actuators. The interconnection of these components is illustrated in Fig. 4.



4: Block diagram of wind turbine model.

III. CONTROLLER DESIGN

The controller design has two main objectives. First of all it must keep the structural/electrical loads within the design specifications. The structural loads are in this formulation measured by the shaft torsion, Q_{sh} , i.e. minimising Q_{sh} will minimise the structural loads in the transmission system. The electrical loads are mainly given by the slip in the generator, and by limiting the variations from a statically calculated generator speed reference the electrical loads in the generator/converter will be limited. The other main objective is the power quality and with the electrical power being proportional to the generator reaction torque, Q_g . The

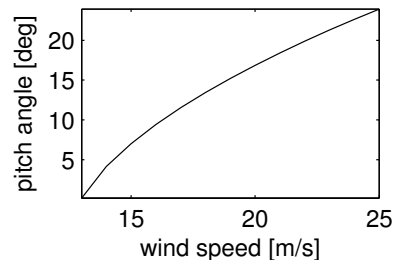
power quality is in this context measured in the variations in reaction torque from a statically given set-point. From these objectives the control formulation is given as a tracking problem of generator speed and torque references and the minimisation of the shaft torsion.

The model described in Section II is highly nonlinear, mostly because of the coupling through the aerodynamics. Also the actuator loops are nonlinear, but in the high wind speed region a linearised model of these loops is deemed appropriate.

It has been chosen to design the controller as a gain-scheduled static state feedback with the effective wind speed as the gain-scheduling variable. Along a selected trajectory of operating conditions the nonlinear model is linearised and an LQ controller is designed to trade off the three objectives described above. The trajectory of operating conditions is determined from the following observations.

A. Target trajectory

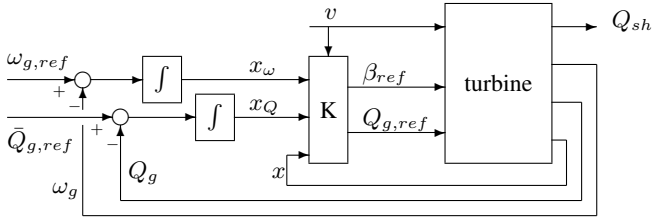
In the high wind speed region the generator speed must be maintained close to a specific rated value in order to keep the electrical loads low. Furthermore the power production must be close to the rated power production in order to maximise the production. The rated rotor speed and aerodynamic torque can then be calculated from the DC response of the linear transmission system model combined with the rated values for the generator speed and generator torque – which is easily calculated from the speed, power and generator efficiency. Then there are only two variables left in (1) and with the assumption that the wind turbine is not operating in the stall region, there is a one-to-one mapping from mean wind speed to mean pitch angle as illustrated in Fig. 5 – in order to obtain rated speed and power.



5: Steady state pitch angles at rated rotor speed and power.

B. LQ design

With the operating points determined, a set of controllers can now be determined using LQ control. In order to have good DC tracking performance for both tracking problems an integrator on each tracking error is included in the setup. This then gives the control setup in Fig. 6 and the controller gain, K , is calculated at each operating point to minimise the cost J_c , in (5). In the implementation the intermediate controller gains are then interpolated linearly from the discrete number of controller gains.



6: Block diagram of controller formulation.

$$J_c = \int_{t=0}^{\infty} z(t)^T Q z(t) + u(t)^T R u(t) dt \quad (5)$$

$$z = [Q_{sh} \quad x_{\omega} \quad x_Q]^T$$

$$u = [\beta_{ref} \quad Q_{g,ref}]^T$$

Remark 1: It should be noted that with the approach of interpolating controller gains, we can give no guarantees in terms of stability and performance for the intermediate operating conditions. In practice, however, the method has shown applicability for several application areas.

One way to overcome the problem is to apply LPV techniques for the gain scheduling. In these methods, the model is scheduled upon a time-varying parameter for which the values are a-priori unknown but measurable online. See e.g. [6], [7], [8], [9], [10], [11].

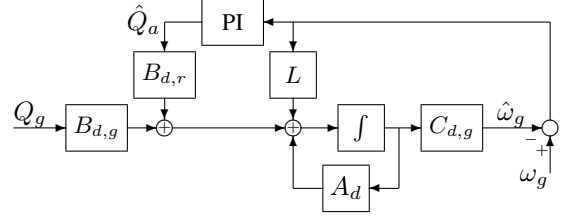
IV. OBSERVER DESIGN

There is no high quality measurement of the rotor speed and only a very rough measurement of the wind speed available. It is therefore necessary to have a good state estimator to get the controllers implementable. Further we need an estimate of the wind speed to get the scheduling variable. A detailed presentation of the observer design is given in [12] and we give only an overview of the parts that directly apply to the current controller design. Preliminary practical experiments indicate that the measurements of the actuator outputs are given with a precision that allows us to focus only on the estimation of the dynamics in the transmission system. This means that we assume that the pitch angle, generator speed and generator torque are online available. We split the observer design into two main components: a dynamic observer for the state and input estimation of the transmission system, and a calculation of the wind speed from the estimated aerodynamic torque.

A. Observer for transmission system

From linear analysis it is clear that the transmission model is observable from the measured output, ω_g . Also one of the inputs, Q_g , is available online – and for simplicity in the algorithm it has been chosen to separate the observer design for the transmission system from the nonlinear aerodynamic model. This means that we must also estimate the aerodynamic input, Q_a . One approach is to augment the transmission model with a state representing the unknown input and use a Kalman filter to estimate the augmented state vector. Alternatively it has been chosen to use a method

where the observer design is split into a standard state estimation problem combined with an input observer. The main idea is illustrated in Fig. 7: A Kalman gain, L , is designed as if the unknown input was available with process noise reflecting the expected variance in the estimation of Q_a . Then an observer is designed in parallel with the observer gain, L , to estimate the “disturbance”, Q_a .



7: Block diagram of observer for transmission system.

It has in this case been chosen to use the PI controller in the disturbance estimation for its simplicity in the tuning process and because it achieves asymptotic tracking. The method can though easily be extended to other controller structures.

B. Calculation of wind speed

In the above sections, a set of observers were designed to estimate the state vector in the wind turbine model. In this section the gain-scheduling variable, v , will be calculated from the output of these observers. Then (1) can be rewritten as (6) with C_{ω} being a constant for each rotor speed and $c_{P,\beta}$ being only a function of λ at a given pitch angle.

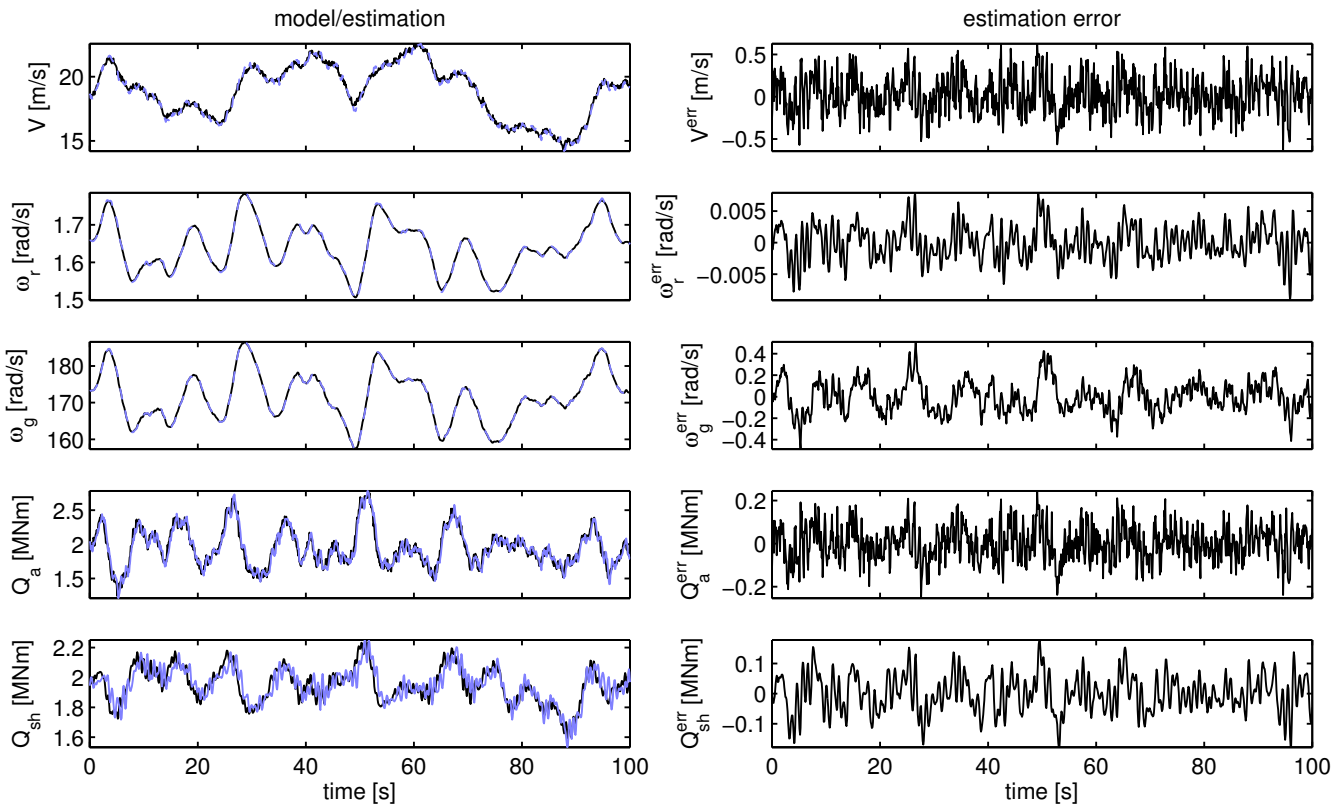
$$Q_a = \frac{1}{2} \rho \pi R^2 \frac{v^3}{\omega_r} c_{P,\beta}(\beta, \lambda) = C_{\omega} \cdot \frac{c_{P,\beta}(\lambda)}{\lambda^3} \quad (6)$$

When the wind turbine is not operating in the stall region, $c_{P,\beta}$ is a decreasing function and thus, Q_a is a decreasing function in λ and therefore invertible. If, however, the wind turbine is in stall operation, $c_{P,\beta}$ is increasing and Q_a is not monotonous and thereby not invertible. From the physical interpretation it can be observed that the phenomenon happens only during stall operation[12], and because of this it is deemed that we will not encounter this problem in nominal operation. Therefore it has been decided to use only the monotonous part of the function in the calculation of the tip speed ratio. When the tip speed ratio has been calculated, the effective wind speed is simply calculated from $v = \frac{R\omega_r}{\lambda}$.

V. SIMULATION RESULTS

The controller and observer design has been validated through simulations with stochastic wind input with wind speeds in the high wind speed region. In this section some of the simulation results are illustrated with wind input reflecting the “A” turbulence described in the IEC norm [13]. It has been chosen to simulate with a mean wind speed of 18 m/s which results in a turbulence intensity of 17 % in order to get wind speeds in most of the high wind speed region.

The simulation model used in the validation of the controller and observer design is of higher order compared to the

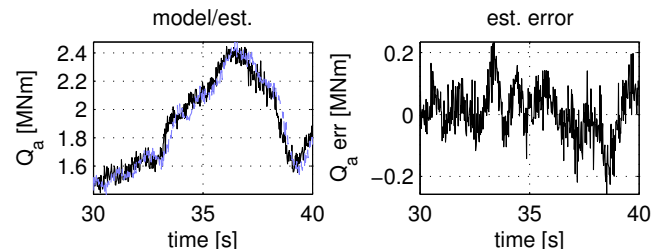


8: Simulation of wind observer at high wind speeds. Blue, dashed lines: model, black lines: estimation

design model. The second order nonlinear model of the pitch system is used, and also high frequency components of the generator model are included. Besides this, tower fore-aft and sideways dynamics are included in the structural dynamics.

Simulation results for the observer for the transmission system combined with the wind speed calculation are given in Fig. 8. The left column displays several variables and their estimated values and the right column displays the estimation errors. The true values are illustrated in black and the estimated values in blue and dashed.

It can be observed that the estimation of the rotor/generator speed is very good. At first it seems that the estimation error in the aerodynamic torque, Q_a , and shaft torque, Q_{sh} , is slightly above what can be expected, but a more thorough investigation shows that most of the estimation error is caused by a small time delay in the estimation and that the torque can change rapidly. An example of this is shown in Fig. 9 and from this kind of investigations it has been concluded that the estimation error is appropriate. Further, a standard deviation in the wind speed estimation of $0.2\text{--}0.3\text{ m/s}$ is deemed small in the context of using it as gain scheduling variable. For the evaluation of the performance of the designed controller, the performance of the newly designed controller has been tested against a controller designed using classical principles that has been validated to satisfy the design requirements on Vestas wind turbines.



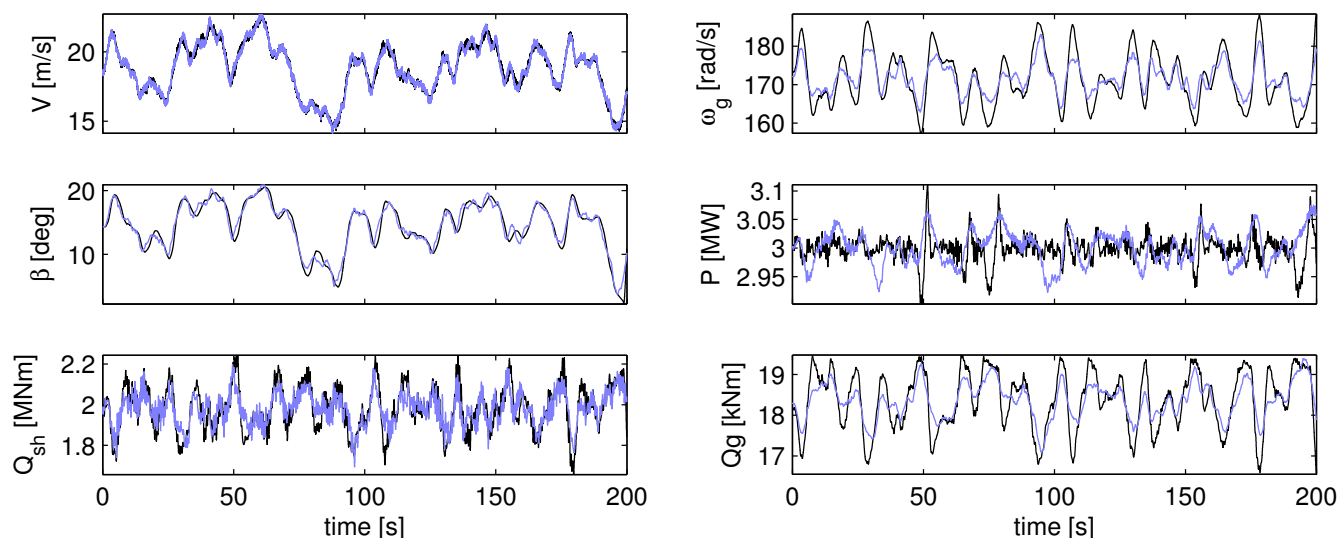
9: Simulation of aerodynamics torque. Blue, dashed line: model, black line: estimation

The classical controller is essentially a PID controller for tracking of the generator speed with the pitch orientation as control signal. The dynamic component on the power reference is a feed forward term on the generator speed band pass filtered around the transmission eigenfrequency. The controller is illustrated in (7) below.

$$\beta_{ref} = PID(s) \cdot (\omega_{ref} - \omega_g) \quad (7a)$$

$$P_{ref} = \bar{P}_{ref} + BP(s) \cdot \omega_g \quad (7b)$$

For the simulations the two different closed loops have been operated on identical wind turbine simulation models and with identical wind input. Simulation results are illustrated in Fig. 10 with the newly designed controller in blue and the classical controller in black.



10: Comparison of LQG (blue) and classical (black) controller.

From the figure it can be observed that the LQG controller is superior to the classical controller in the sense of fatigue loads because it has significantly smaller variations in generator speed for similar shaft torque. Furthermore this performance is obtained for similar control effort in the pitch system and less effort in the generator torque.

From the graph of electrical power it can be observed that the LQG controller has slow variations around the nominal power level of 3 MW. The classical controller has in contrast much smaller variations except from a few small time intervals in which the fluctuations exceed the level of the LQG controller. From this it is concluded that the classical controller has better performance when observing the power quality.

The difference between the two controllers is caused partly by differences in the tuning process. It should, however, be noted that there is one significant difference between the two controllers in that the LQG controller does not contain the power as a variable. The reason is that power is not suitable as a variable for linear controller design because of the nonlinear coupling between torque and power. This means that the power can only enter in the performance criterion and not in the controlled channel making it more sensitive to under-modelling. In the classical controller this is not an issue because it is tuned on the basis of the nonlinear model making it possible to include the power as a variable in the controlled channel.

VI. CONCLUSIONS

This paper addressed the problem of designing a gain-scheduled linear quadratic controller combined with a state and disturbance estimation algorithm. The controller was designed by linearising the nonlinear plant model along a trajectory of operating points scheduled on the effective wind speed. The observer was designed by a modular approach in which a linear observer was designed for state

and disturbance estimation in the transmission model and the wind speed was calculated from inversion of the static aerodynamic model.

The simulation results showed good performance of the observers and the comparison of the resulting closed loop system with another controller designed using classical methods showed good performance in terms of fatigue loads. This good performance came with the cost of slow variations on the active power.

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